MUSIC, MATHEMATICS, AND THE CYCLE OF FIFTHS

An excerpt from

THE TOTALITY OF GOD

AND THE IZUNOME CROSS

Unlocking the Secret to the Riddle of the Ages

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Music, Mathematics, and the Cycle of Fifths

In the previous essay, *The Mechanics of Spiritual Healing*, I reintroduced a balance that in the main text had made a brief but highly significant appearance in the *Theory of Harmonic Creation* – Music and Mathematics. You'll recall I noted that the *System of Quadrality* is based on the balance of word-energies, and these reflect the two Root areas in which harmonics have relevance. Their truth is so universal it could be applied to areas as divergent as wave motion, music theory, and even, as in that essay, spiritual healing.

On p. 754 I explored how Music and Mathematics unify through Physics, and a portion of its text is worth repeating:

> Then, "harmonic" takes on the ability to describe a periodic vibration, or back-and-forth motion, in which such motions are symmetrical about a region of equilibrium. . . . Thus, it is through Physics that terms like frequency, associated with Music, blend with terms like inverse, associated with Mathematics. Physics even merges Music and Mathematics in its definition for *Fundamental*: "the lowest frequency of a periodically varying quantity or of a vibrating system." And they all come together through *Harmonic Creation* to produce a motion that could as easily be used to describe the vibrations of sub-atomic particles as those of a violin string. I labeled their merging as perfected to reference the Physical Perfection of God that the *Theory of Harmonic Creation* serves. The laws that govern either motion are one and the same, and it is this truth that underscores their movement. However, it is the ultimate *Truth within the Movement* to which the *Theory of Harmonic Creation* speaks.

This balance, so essential to the truthful movement of the Universe, will later help us understand the mechanics of multidimensionality. But it was in the main text where we saw just how dramatically the beauty and precision of the Universe, unified through music and mathematics, were revealed in the resonant frequencies of the Great Pyramid of Giza and the four base elements of DNA.

However, when the formal definition for the *Theory of Harmonic Creation* was given on p. 399, it was not the first time the balance of music and mathematics had appeared and been addressed. That occurred in the few pages immediately preceding Figure H-U, beginning on p. 368. But I must admit that in the original draft for the *First Print Edition* those pages had been somewhat different. Though I'd pursued music to examine the mechanics of harmonics as they manifest through sound, I had dealt more with the physics of scale construction and less on the underlying truth as it would later be revealed to me through my *Theory of Harmonic Creation*. I'd yet to even fully grasp it, let alone define it. And what that theory would eventually lead me to – the evidence for the *System of Quadrality* in the Great Pyramid and DNA – was nowhere in my conscious mind. Thus, the reference to that discovery in *Footnote 150* was obviously after the fact, and *Footnote 149* likewise was substantially different. Dear Professor Einstein's dream of a *theory of everything* had yet to cross my path.

So, as has happened more times than I can count in the revision process, something I'd written had led me to the truth I was meant to see, but the writing itself had served more as a place holder for what I'd eventually need to include there concerning it. In fact, those pages contain the last revisions to the main text, and I'd originally been drawn there simply because I needed some space to offer a definition for the *Spiritual Principle of Related Frames of Reference*. In all the 1000 pages of this work, only p. 371 at the time contained sufficient space! Of course, it came as no surprise to me that the location was also the perfect place to put such a definition. But more significantly, it caused me to reexamine those few pages preceding it. And I realized I'd missed in the original text the opportunity to consider the deeper truth revealed through the balance of music and mathematics. It took all the editing skill I had to present in that revision what I needed to in the space allotted. And yet, I felt I'd slighted the subject, and cheated my readers, as a result.

That explains the reason for this essay's appearance – to rectify the inevitable oversight. But as to the title, that is another matter. Becoming a truthful musician requires any student to absorb at least the basics of music theory. Chord construction is required for harmony, and a scale, for melody, the two of which in the aforementioned revision are acknowledged as the vertical and horizontal principles in music. We have associated these event-lines with the cyclic and linear principles of space and time before, and will do so again. But in music, harmony and melody become a true quadrality through cyclic and linear. A melody is cyclic in the way portions of it can repeat, and harmony is linear in that notes don't have to be played together to create it. These principles occur within the frame of reference of a specific key for melody and harmony. Western 12-TET tuning, as explained in *Footnote 149*, concerns a musical Universe with twelve keys forming the birthplace and home of its frequencies and harmonics. Yet these keys have a profound relationship to each other, which can likewise be understood through the principles of cyclic and linear. The linearity of keys allows musicians to modulate from one to another in the course of a single musical piece. Their cyclic nature makes it possible to transpose an entire song into a new key, start to finish. Both of those abilities arise from the beauty and precision of music's naturally occurring harmonic structure, first explored scientifically by Pythagoras over 2500 years ago. It is he that is credited with discovering the mathematical relationships between frequencies that sound pleasing to the ear. And when his simple ratios were then applied to create a series of notes – each one following in proportion to the previous – the result was a scale similar in form to those that are still in use today. The process of applying the same *Pythagorean* ratio, 3/2, to generate the series of notes became known as the *Cycle of Fifths*, an essential truth within the movement of the entire Universe.

Why that is so we'll soon discover. But I'd first become fascinated with the *Cycle of Fifths* while studying music, which I'd pursued mainly because it was never enough for me simply to copy a bass line someone else had played. I had to understand what had allowed them to create it. The *Cycle* was intriguing not just because of the way the keys seemed to evolve around each other, but also the way all the sharps and flats associated with those keys did likewise. I'll soon provide a graphic demonstration of its beautiful precision, a reflection of the history of musical truth learned throughout the ages – which we'll then discuss as it was both revealed and applied. And quite amazingly, the realization of that truth did not begin with Pythagoras. It is as inherent in our being as are the resonant tones of our DNA. Pythagoras merely formulated the earliest mathematical relationships for it. But while doing research for the revision, I learned that a flute had been found dating back to the time of the Neanderthals – about 50,000 years! And some argue the spacing of its four holes would have allowed for the playing of four notes in the diatonic scale!!

Throughout this work, for the bulk of my research into definitions and general subject matter I've used Microsoft's *Bookshelf* and *Encarta 98*. Actually, I began with the *Encarta 95* version, and a few of its references have remained for reasons of content. *Bookshelf* itself compiles other reference works, including the *American Heritage Dictionary of the English Language* and the *Concise Columbia Encyclopedia*, both listed in my bibliography.

In the technical areas this information was substantially added to from my personal copies of the high school and college text books I had underlined and sweated through – *Modern Phy*sics, and *Physics Part 1* and *Part 2* – also credited in the bibliography. The fact that these books are 40 or more years old matters not; while we've increased our knowledge of physical truth, its underlying mechanical laws, courtesy of geniuses like Sir Isaac Newton, have largely remained unchanged from their discovery over the past 300 years. So, I would not be surprised to see copies of these works still floating around classrooms today. *Modern Physics* even has a wonderful chapter on music, "Musical Sounds"; and much of the technical information on scale construction and music history in the main text had come from it.

But I learned of the flute discovery through a marvelous on-line reference library, which I'd come across near the end of this entire writing journey. It is called *Wikipedia*, and it is unusual in that it is the work of the people who actually use it, fully editable by anyone knowledgeable on a particular subject. Hence, none of the articles are credited, and the same information often appears in several, as happened with the Neanderthal flute. *Wikipedia* has a wealth of articles related to music, with cross-referencing links that can have you searching for hours; and what I learned while doing so inspired me to write this essay in which I could pursue the subject more thoroughly. It was mainly the mention of the *Cycle of Fifths*, and the instant recall of the interest I had in it decades ago, that sparked the fire. So, I want to state my appreciation to *Wikipedia* up front and reference it collectively in the bibliography, as it would be futile to mention the articles used in this essay individually. Nothing will be quoted directly, and the chart for the *Cycle of Fifths* is my own. But for the next few moments, I will supplement the insights into the story of music gained from my personal experience or from any other sources specifically mentioned by sharing with you the essence of what the folks at *Wikipedia* shared with me.

So, what is the *Cycle of Fifths* and how did it come to be? And more significantly for us, what does it mean? What truth does it reveal about the Universe's movement? To really appreciate it we must realize that music's mathematical journey, begun by Pythagoras, wasn't an artifice separate from music but an inherent part of it. It had been there all along, merely waiting for its discovery. And whatever contrivance mathematics would later provide music to work around what we'll next see is its intrinsic paradox, was only to find a way to balance Order and Chaos, just as everything in the Universe, and the Universe itself, has to for survival.

In arriving at his discoveries, Pythagoras was studying natural phenomena. His mind tuned him into their precision. Pythagoras saw the mathematics in the relationships that euphonious notes had to each other, but what made them sound pleasing to begin with was imbedded in our consciousness, as it is in all nature. The specific notes were not important, only the proportional difference – or ratio – in their frequencies, which would establish the pitch distance – or interval – from one to another. These are the qualities that are important to the human ear, and as we saw with the Giza/DNA analysis, the relationships that allow the Universe to exist at all. A singer or musician needs to have a good sense of relative pitch – to be able to discern the intervals and ratios – without needing to have absolute pitch – knowing the exact tone value upon hearing it. There are some who have the latter gift, but most, like myself, get by very well with the former.

Pythagoras discovered that the most pleasing interval was the octave, which, no matter the tonic, will have a frequency ratio of 2 to 1. The second most pleasing interval was the fifth, called "perfect," having a ratio of 3 to 2. The *Cycle of Fifths* then grew from the understanding that if you start at a note, say C, and use this ratio to find its fifth, in this case G, the same ratio could then be applied to G to find its fifth, D, and so on. Moreover, you could go in the other direction from C with the realization that it itself was the fifth of another note, now F, found at a ratio of 2 to 3. Again, F is the fifth of B*b*, and so on.

The great paradox comes when you follow this to its ultimate conclusion. Were you to pursue this mechanically on any modern western instrument with 12 fixed pitches between octaves, like a guitar or piano, you will eventually come back to C whether you go forward or in reverse. Yet, if you pursue the pitch values from the purity of the Pythagoras 3/2 interval, you will never complete the loop! To illustrate, I'll now provide my chart for the *Cycle of Fifths*. There is much more for us to learn concerning this, but it will be best to have the *Cycle* now for reference as we do so.

375.89	↑	F#	6	F#	C#	G#	D#	A#	E#
501.19		B	5	F#	C#	G#	D#	A#	
334.13		E	4	F#	C#	G#	D#		
445.50		A	3	F#	C#	G#			
297		D	2	F#	C#				
396		G	1	F#					
264		C	0						
	3/2	**Key**	**#'s**						
Frequency (hertz)									
	2/3	**Key**	***b*'s**						
264		C	0						
352		F	1	B*b*					
469.33		B*b*	2	B*b*	E*b*				
312.89		E*b*	3	B*b*	E*b*	A*b*			
417.19		A*b*	4	B*b*	E*b*	A*b*	*Db*		
278.12		D*b*	5	B*b*	E*b*	A*b*	D*b*	G*b*	
370.83	↓	G*b*	6	B*b*	E*b*	A*b*	D*b*	G*b*	C*b*

<u>Figure C-F</u> : The *Cycle of Fifths*.

Even if you don't know the theory behind it, its symmetry leaps off the page! In the upper half of the diagram, the *Cycle* moves forward from C through the keys. And as it does so, each key brings with it an "accidental," a sharp note not in the C diatonic scale of C, D, E, F, G, A, B, and C octave. With each new key an additional sharp gets added to the ones previous to complete its key signature (the aggregate of sharps or flats that identify a key). And these sharp notes themselves follow their own 3/2 *Cycle of Fifths* in the forward direction.

In the lower half of the diagram, the converse happens as the *Cycle* moves in the reverse direction from C through the keys. As it does so, each key likewise brings with it an "accidental," in this case a flat note not in the diatonic scale. And with each new key an additional flat gets added to those previous to complete its key signature, with all of these following their own 2/3 *Cycle of Fifths* in the reverse direction.

In either direction I have ended the *Cycle* with 6 accidentals, 6 sharps for F# and 6 flats for G*b*. On a 12-TET instrument these keys are mechanically equivalent. Below G*b*, keys are best served, and already are, by the sharp keys. Above F#, keys are best served by their flat equal-temperament counterparts. Moreover, the final accidental added in each case is actually an *enharmonic* designation for a note in the diatonic scale, being identical in pitch but written differently according to the key in which it occurs. E# is equivalent to F, and C*b* is equivalent to B. But with all this talk of equivalence, the paradox of the frequency values in the first column is even more pronounced. F# and G*b*, which are equivalent from the mechanical perspective of 12-TET, are not frequency equivalent from the perspective of being derived through the *Pythagorean* ratio. (To arrive at these values, I simply used a calculator and began with 264, multiplying by 3/2 in the forward direction and 2/3 in the reverse. Frequencies were then resolved when necessary, through division by 2 in the forward direction or multiplication by 2 in the reverse, to arrive at frequencies in the same octave. Lastly, all results were rounded to the hundredth decimal place.)

You may then question why 12 keys became the modern standard, as if it were a stratagem not connected to the reality of nature. But *Wikipedia* reacquainted me with something amazing, first brought to my attention in *Modern Physics*. 12 is a natural truth in any key's relative frame of reference that mechanics has to adjust to accommodate moving between keys in the Universe's absolute frame of reference, or even for getting beyond the diatonic scale in a given key.

To understand this, let's go back to where we left off with Pythagoras and his intervals. I first mentioned the diatonic scale on p. 368, saying it was perhaps the most fundamental in western music and best illustrated by the white keys on the piano. The names for those white keys are the very same just listed as the C diatonic scale. And by looking at Figure C-F we can see why we have Pythagoras to thank for them. Six are found by going forward through the *Cycle of Fifths* from C, and the seventh by going once in reverse. Clearly, G is the most significant mathematically, being the first after C going forward. And it happens that when the audible overtones of C are considered, G is also significant, with only the C note itself being more audible. But the next most significant note in the diatonic scale from the perspective of mathematics is not found by continuing to go forward. Instead, it is the first found by going in reverse, F. While G is the 5th to C, the tone center for this diatonic scale, C is the 5th to F. That is, in the key of F, C is its 5th, at a ratio of 3 to 2 from F. But we arrived at F from C, which gives F the next most significant position in the key of C following G, that of being its 4th. And the name given to the 4th in any key expresses this significance, the subdominant, with the 5th, as mentioned in *Footnote 150*, the dominant.

Why am I going into such detail on this? It is so you grasp just how important the existence is of simple mathematical ratios in what the human race inherently understood since its earliest times as pleasing sounds. The octave is found at a ratio of 2 to 1, the 5th at a ratio of 3 to 2. The 4th is then at a ratio of 4 to 3, which is confirmed by how I arrived at F in Figure C-F. I had to multiply the frequency of C by 2/3 and then multiply that by 2 to place it in the correct octave. These three intervals, 1st, 4th and 5th, are the foundation of any given key, found in the music of virtually all cultures throughout the ages. And for any melody played from the notes in a diatonic scale, these chords alone will suffice to provide its harmony. That is, in the key of C you don't necessarily need some form of A chord to harmonize an A in the melody. The F major chord, containing A, will often work.

It's through the chords accompanying a melody that harmony integrates with it. And as *Wikipedia* explained, all this happened quite naturally during the course of human history. We've just seen why the chords formed from the three intervals, which musicians often refer to as the one-four-five progression, are essential to any key. But the notes that complete the diatonic scale have more than the mathematics of the *Pythagorean* ratios to thank for their existence. They have their own harmonic nature. Several articles mention a "trio theory," formulated in the mid 20th century, which indicates the diatonic scale grew out of the naturally occurring overtones of the three notes established by the three *Pythagorean* ratios: 1st:2/1, 5th:3/2, and 4th:4/3. With the root and 5th being the most audible overtones, the next is the major 3rd. This interval was mentioned on p. 368 as having the frequency ratio of 5 to 4 with respect to the root, and I'll further explore it in a moment. But for now, consider how the 1, 5, 3 overtones for C produce C, G, E, the C major chord. The overtones for F and G produce F, C, A, and G, D, B, the F major and G major chords, respectively. When these notes are resolved into the same octave and duplicates are eliminated, you arrive at C, D, E, F, G, A, and B, the C major scale, which contains the C pentatonic scale, C, D, E, G, and A, as well as the A relative minor scale. This is the C diatonic scale begun at its 6th interval. This relationship between a major key and its relative minor key is extremely important, and was instrumental in helping us to understand the balance between consonance and dissonance in the four fundamental frequencies resonant in the Giza Pyramid and base elements of DNA. But it is in this interval, the 6th, where the harmonic truth of the relationship between major and relative minor keys disengages from the mathematical truth of Pythagoras. And it occurs, not because of the relationship any of the root notes for the three chords has to its 5th, but to that of its major 3rd. The significance of the 3rd interval, also referred to as the mediant, has already been shown in *Footnote 150*. In the four chord-forms upholding the four realms, whether the 3rd is major or minor established if it upheld the Extremes or Means for the Universe. And while I didn't mention it then, the dominant 5th, the same in both spiritual chord-forms and different in both physical chord-forms, seems to uphold the Root Realms as *Nonchange* vs. *Change*.

The truth of 12 tones in any key can now be resolved harmonically. The next most audible overtone after the major 3rd is the flat 7th. And if you then gather the overtones for the seven notes in the diatonic scale, you will arrive at the 12 in the chromatic. Not all duplicate tones will match frequencies exactly, which is why I said adjustments had to be made even in one key to go from the diatonic to the chromatic. But they are sufficiently close for tuning compromises to establish 12 keys.

Modern Physics also derived the diatonic scale through simple ratios. But then, they were arrived at not as a result of analyzing overtone intervals, but through a mechanical device known as a sonic wheel. Without going into great detail on it, the result was that the tones of any major chord could be shown as exhibiting vibration ratios of 4, 5, and 6. The entire diatonic scale could then be expressed as simple ratio relationships from the tonic. We've already covered four of them. The following chart will reveal the remaining three and reference them to their respective notes in the C diatonic scale, as well as to their interval from the tonic. Below them are listed the frequencies derived from these ratios (as well as the Figure C-F frequencies gotten from the three *Pythagorean* ratios, now rounded to the nearest hertz for ease in comparison). When a scale is tuned using these simple ratios, it is referred to as *Just* intonation, and thus the reason for *Just* being associated with the names for the ratios as well as the frequencies derived from them:

Diatonic Note:	C	D	E	F	G	A	B	C
Interval:	1ST	2ND	3RD	4TH	5TH	6TH	7TH	8TH
***Just* ratio:**	**1**	**9/8**	**5/4**	**4/3**	**3/2**	**5/3**	**15/8**	**2/1**
***Just* frequency:**	**264**	**297**	**330**	**352**	**396**	**440**	**495**	**528**
***Pythagorean* frequency:**	**264**	**297**	**334**	**352**	**396**	**446**	**501**	**528**
***Pythagorean* ratio:**	**1**			**4/3**	**3/2**			**2/1**

First, to clarify, the 8th interval, the C octave, has the frequency of 528 whether it is *Just* or *Pythagorean*. The wave equation (p. 360) fixes that as mechanical truth, with frequency doubled when wavelength is halved. Pythagoras set 2/1 for the octave as mathematical truth. But if you follow his 3/2 interval through the *Cycle of Fifths* going forward, you arrive at a frequency of 535! So, his system's inherent flaw, and resulting enharmonic discrepancy (p. 780), called the *Pythagorean comma*, is clear. Yet, *Pythagorean* ratios are quite valid, producing frequencies identical with the *Just* ratios for the 4th and 5th, which thus likewise unify mechanical and mathematical truth. Indeed, the 2nd for these systems are also identical. Only in the 3rd, 6th, and 7th do frequencies deviate, with the 3rd appearing before the others in the overtone series. Hence, even with the 1st and 5th the same, any chord containing the major 3rd would sound slightly different depending on the system in use. And since the 6th is also different, which appears before the 7th in the overtone series, the pitch of the relative minor key for the *Just* ratios would similarly not conform to the *Pythagorean*.

Nonetheless, *Just* intonation grew out of the desire to make the scale sound right rather than conform strictly to the *Pythagorean* version of the *Cycle of Fifths*. And there is an extraordinary relationship that these ratios have to *prime numbers*, mathematical truths that, as we'll much later learn, are at the foundation of the Universe's structural and harmonic integrity. It happens that the *Just* ratios can all be formed as factors of the first three primes, 2, 3, and 5. For instance, the 7th *Just* ratio 15/8 = 5/2 x 3/2 x 1/2. However, *Just* intonation wasn't without its own inherent flaw, which became evident once musicians tried to apply the same intervals to the notes in the C diatonic scale, using them as tonics to create diatonic scales in these new keys. Some notes would match, but as *Modern Physics* showed for the D diatonic scale, others could differ by as much as nearly 30 hertz, a difference so drastic they were given new note assignments as #, and thus not in that diatonic scale! In this way, arriving at the diatonic scales for all notes in the C major scale, the C chromatic scale was formed. A page ago I explained how this could be done harmonically with only the first four unique overtones of each note in the diatonic scale. But no matter the road you take to find them – the *Cycle of Fifths*, *Just* ratios, or overtone series – these enharmonic accidentals may be mechanically equivalent but not frequency equivalent. Each of the five accidentals found in the C chromatic scale, matching in 12-TET the black keys on a piano, could have a different frequency value depending on if the sharp or flat enharmonic is called for. Thus, *Modern Physics* provides 17 pitches for the C chromatic scale given this possibility, indicating an instrument would need about 70 notes between octaves if all twelve keys were being built in this way!

The evolution of music as a performable art form largely became a quest to resolve this paradox through systems and instruments based on them that would allow musicians to not only navigate between keys, but merely even play complex melodies and harmonies in one key. *Meantone* temperament, popular up to around the 1500's, constructed scales as a chain of perfect fifths but then adjusted the intervals in favor of the major third, making them closer to their *Just* ratio. In the

1700's, *Well* temperament moved away from the *Just* ratios by adjusting all intervals so that no key would sound perceptibly out of tune. But as a result, each key was said to have a distinct characteristic, referred to as its key-color; and despite the fact that modern tuning has eliminated interval distinctions, many musicians to this day make this color distinction, preferring how a song sounds in one key over another.

The quest to resolve the tonal paradox essentially ended in the early 20th century when the 12-TET scale was adopted for the pianos found throughout the world in places ranging from living rooms to concert halls. It is an international standard that I earlier referred to as western, but only because it seemed to evolve primarily in a western culture whose musical tastes and needs fostered it. Other cultures' tastes and needs seemed to require less conformity in tone or structure. And there are many diverse forms of world music, with its musical instruments and musicians who play them. Some modern western composers still write for one of the earlier systems, or even for their own, in which regard electronic instruments have made significant contributions. But in general, 12-TET fixed-pitch instruments uphold the predominant position, especially when universality or commercialism is sought. Hence, we will now turn our attention to the musical truth within its mathematical movement.

In the early days of attempts to arrive at a sonic compromise, efforts focused on nudging the notes to concentrate on the most used intervals, the 3rd and the 5th. The ultimate judge was the musician's ear – what sounded good. The change occurred as musicians began to consider smaller intervals, the smallest of which on a piano is the half-step – simply going up the chromatic scale from one note to the next. The practice of tuning to create equal sonic divisions between the two notes that establish any octave became known as *Equal* temperament.

I find it interesting that musicians didn't just automatically jump on the 12-TET bandwagon. Variations were tried, including some you might think of as strange, such as 5, 7, 19, 31, and 53, all, interestingly, *prime numbers*. Others were harmonic subdivisions of 12, with 24 and 72 being popular examples. Both types will be worthy of our considering from the perspective of Universal Truth. But as mentioned earlier, 12-TET was chosen because it contained the fewest divisions required to approximate in all keys the most important intervals: perfect, major, and minor. These were the ones that in the series of overtones were the strongest, or most audible.

Essentially, *Equal* temperament continued the process of key compromise and interval homogenization begun in *Well* temperament by taking an actual *Cycle of Fifths* but then narrowing each interval by the same amount. This isn't as easy as it may sound because the difference between pitches is not linear. Consider how the frequency difference between each successive octave doubles. Middle C is 264; C′, the first octave, is 528; C′′, the second octave, is 1056. *Wikipedia* explains that true *Equal* temperament wasn't possible until Hermann Helmholtz published a detailed study of acoustics in 1863. Even so, then making the dream of *Equal* temperament a reality required a logarithmic solution.

Until now in this essay we have been speaking of scale in a musical sense, as a structured series of tones varying in pitch arrangement and interval size. But we must also consider scale as it applies in a mathematical sense. *Bookshelf* defines it as "a system of notation in which the values of numerical expressions are determined by their places relative to the chosen base of the system." In the main text, of these two uses music got the most attention, with the mathematical application of scale noted only on p. 377 in *Footnote 153*. That use will continue to receive greater attention in upcoming essays, but it is important to begin here in a context that unifies them. The previous essay's *Footnote 2* gave a brief mathematical explanation of the difference between an arithmetic and a geometric sequence, focusing then on the arithmetic. Both are best grasped through the means used to graphically depict scale, and I won't even have to draw them; a simple explanation will suffice. It is common to graph a set of values on the X-Y axes of a Cartesian coordinate system. When equidistant points on either axis have the same difference in value, the sequence represented is arithmetic. With 1, 2, 3, 4, etc., the difference is 1. With 5, 10, 15, 20, etc., the difference is 5. Simple enough, and both frames of reference have a relationship to one another through a constant. The second sequence can be arrived at from the first by multiplying each value by 5. This reflects a difference in proportion, or scale, which would allow data plotted in one frame of reference to also be plotted in another, with less or more distinction being value-dependent. For instance, fifteen points between 6 and 7 on the first frame would be much less distinguishable on the second.

However, there is a second graphic application, one where equidistant points don't increase in an arithmetic sequence but in a geometric one. As an example, 2, 4, 8, 16, 32, etc., is a geometric sequence where the difference between values in each case doubles. Yet, both types of sequences apply to music. The series of harmonics, which is often referred to as the harmonic series (though we must not confuse this use of series with its arithmetic use as outlined in *The Mechanics of Spiritual Healing*), is my first arithmetic example. Given any fundamental, multiplying it by 1, 2, 3, 4, etc., will produce its sequence of harmonics. However, as we just showed, the evenly spaced octaves on a piano are a geometric sequence – the frequency difference between each doubles. The arithmetic example is linear while the geometric one is logarithmic. But it happens that our ears hear the pitch distance between linear harmonics as decreasing while we hear the pitch distance between logarithmic octaves as the same. In other words, we hear sound logarithmically!

This explains what may have seemed curious re the overtones of "trio theory." The 1st, perfect 5th, and major 3rd are the most audible overtones of a fundamental. And yet, when harmonics are created as subdivisions of a fixed vibrating string or column of air in an ideal system, only multiples of that fundamental are produced! But because we hear logarithmically what was generated linearly, these multiples are heard as new pitches; and as a result the first 31 harmonics of any fundamental will produce the notes required for the 12-TET scale, granting its tonal compromises.

Several important observations can be made once you include information that helped us draw conclusions in the Giza/DNA frequency analysis. The above harmonic and logarithmic truth corresponds to the mechanical truth revealed in the quote on p. 489 from the George Wedge study of ear training. When a string is mechanically divided into sections corresponding to the higher harmonics, plucking either section of a string divided in half – its 2nd harmonic divisions – produces its octave. Plucking the 2/3 section of a string divided into 3, its 3rd harmonic divisions, produces its 5th. (And yes, dividing a string by 2/3 to arrive at the same frequency as when the fundamental is multiplied by 3/2 does seem to reflect behavioral reversal.) Moreover, as Mr. Wedge mentions, this process of string division can then continue with each 5th used as the fundamental length to generate the next 5th, and so on until the diatonic scale is formed.

Another point is even more significant for us. Once you arrive at 16 harmonics for the three intervals at the foundation of any key, the 1st, 4th, and 5th, you have all the pitches required for the Universe to balance Order and Chaos in the two chord-forms found in DNA, the minor 6th and half-diminished 7th. In the key of C they are

C, E*b*, G, and A. In DNA they are F#, A, C#, and D#. And I believe that in any fixed material frame of reference found throughout the Universe, these intervals will be upheld in whichever key is required for that frame of reference at that time. In fact, the Earth's may have been different much earlier in its history. We can only speak of what it is now. Yet, with an infinity of frequency choices, even in a variable frame of reference the right ones can be found at a given place and time to balance structural integrity with vibrational truth. Even the slight variances we found from our 12-TET reference in analyzing DNA make total sense now that we grasp the intricacy of the systems that must be integrated and the inherent flexibility the Universe has to do so.

But there is a third realization that is perhaps the most powerful of all. When you climb the great pyramid of universal diversity and arrive at its apex, at Unity, at God, you can understand how all we see, and all we can't, could have come from a single frequency – what Susan Alexjander had noted Dr. Larry Dossey as calling the "Great Tone." And it is Pythagoras's *Cycle of Fifths* that we can again thank for this truth. We saw that when you follow it around to complete the *Cycle*, you don't return to where you began. An excellent on-line source describes a mathematical proof for this, which, as paradoxical to me as is the *Cycle* itself, is offered as a valid proof by contradiction. It is found at a link in "Explaining the Equal Temperament," by Yuval Nov, and in the article he surmises it would require an infinity of notes in the range of a single octave. [http://www.yuvalnov.org/temperament/index.html] It's a variation of Zeno's Paradox, *Infinity Within* – between two points – versus *Infinity Without* – beyond them, as I described on p. 171. And my explanation then for space and time displacement we will later apply to rotation. But here, we can see why I was able to say on p. 449 that all frequencies could have been born from a single frequency in the first harmonic. That is all it took to create an infinity of each. It is not my conjecture, but a *Truth within the Movement* of the Universe.

The conclusion one can draw in my making that last statement should be clear. This is a truth for everything, and everybody, everywhere. This is not just for the Planet Earth and its inhabitants, but for any other celestial body and whatever sentient beings may reside there. An important plot point in the movie *Contact* is that mathematics is the only universal language. Thus, when aliens try to communicate with us it is their chosen means for doing so. But if you think about this, if we were to show a visitor from the cosmos the number 3 or any earth-word for it, or if they did the same for us with their numerals or words, would either immediately have the slightest idea what was being communicated? Wisely, the aliens send a calling card in the form of noise-pulses arranged into groupings that represent the *prime numbers*. How the message next sent translates into a machine capable of intergalactic travel is where a viewer's suspension of disbelief is required. But in their calling card lies the truth of the universal recognition of primes and their importance in the *Big Picture*, as well as the ability of sound and rhythm to convey them.

Still, we should replay another movie to see where *Contact* itself lost sight of half the *Big Picture*. Who can forget the means of communication in *Close Encounters of the Third Kind*? They were perhaps the most memorable 5 notes in movie history, adding the octave below the root to 4 of the 5 notes of its pentatonic scale! And the final interchange between races sounded like a cosmic symphony out of control. Disbelief suspension required? Of course! But consider the inherent truth revealed. Without needing to know what a C note is called, simply playing 264 hertz relays information common to anyone capable of receiving it, which in the context of other notes can invoke many basic principles of mathematics and physics!

So, while the information needing to be conveyed is mathematical, music may be the best means of doing it. The truth of frequencies and the harmonics that generate them needs no translation. Moreover, the symphony hall of space and time in which they are played is universal. And I want to leave you with this final thought, courtesy of yet another movie, *Mr. Holland's Opus*. As Mr. Holland comes to learn while leading a concert for his deaf son's classmates, hearing isn't required to perceive the truth in the music. It is found in its rhythm and vibration, and extends to levels of perception often missed by the human ear. For, once you separate music from its emotion, that which is born in the heart of its creator, what are melody and harmony but mathematically arranged vibrations in space and time to which any receptive sounding board can respond. The Tacoma Narrows Bridge didn't have to be able to hear to fall apart from the wind singing to it, nor were evil intentions in that song. But when there are, imagine how much more destructive such vibrations become at perceptible human levels. Thus, take care in the music we listen to and create. And this caution isn't meant only for the music lover or musician. Those vibrations exist in each thought, word, and deed of every player in the orchestra of life.

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This is being written as a postscript merely because I couldn't find an appropriate place to insert it. I didn't want to interrupt the flow leading to the final philosophic conclusion, or diminish its impact by directly attaching this to it. And you will need some of the prior information for this to make sense. One thing we must surely not overlook is how the harmonics of music, the proportions of mathematics, and the physics of mechanics all uphold one another. Consider, for instance, how this occurred for the mechanics of the sonic wheel, the *Pythagorean* ratios, and the overtones of trio theory, with all arriving at the same 5th interval – as well as its inherent paradox. And now that we understand the relationship that mathematical logarithms have to musical truth, I can expand on the point made at the end of *Footnote 149*. It is a remarkable connection between music and mathematics that, like much of its section of the main text, I hadn't been able to do justice. I then mentioned I found it rather extraordinary that "the 12-TET logarithmic truth of a piano's keys is reflected in the mechanical truth of a guitar's frets such that the 12th fret divides a string perfectly in half." Actually, the parallel goes even deeper. Logarithms establish proportional relationships between terms in a geometric series. In the 12-TET scale, the ratio between two adjacent frequencies, called semitones, is about equal to 1.0640309, arrived at by deriving the 12th root of 2. In other words, it is the value that if you multiply any initial frequency ***f*** by it 12 times, you'll arrive at 2***f***. This ratio is not only responsible for the pitch of every key on a piano, but for the placement of the frets on a guitar that then allow those pitches to be played upon it. The width between adjacent frets increases by this ratio for each lower note!

Now, I must be clear that a logarithmic value could be found for any number of intervals into which you might choose to divide an octave. 12-TET was not the only tempered scale considered, and the interval for any other would simply be that root of 2. We have already explored why 12-TET was chosen, and why there is harmonic concurrence with that decision. But such corroboration exists on a guitar in a way that profoundly connects music, mathematics, and mechanics. It occurs through a phenomenon known as tap harmonics. Of course, harmonics are generated whenever any fundamental is played, regardless of it being on an instrument with fixed pitches or on one where pitch creation is flexible, such as a violin, on which, as Yuval Nov points out, pitches can be considered infinite. While in keeping with what I've earlier said, infinity then becomes subject to the limitations of the human ear. However, on many instruments it is possible to generate higher harmonics without playing the fundamental. On wind instruments it is called overblowing; a brass player does it through lip vibration. Since the 1960's, electric guitarists have explored them using various picking techniques. But its orchestral applications were long known, and some classical pieces for violin had such notes written into them, extending the range of the violin even beyond its already high limits. These harmonics are located at specific places along the string, namely the nodes of the harmonics in the overtone series. You should recall that these are the points in a standing wave where there is no movement, and those points will change depending on the harmonic being generated. Moreover, they occur at places dependent on the length of the string producing a given fundamental. So, when two or more strings with different values for diameter, density and tension, but of equal length, are placed together, harmonics occur on all at the same points. It happens that on a fretted instrument, many are over frets! Musicians often use matching overtones found at different frets on adjacent strings to check their tuning. And tap harmonics, mentioned earlier, is a technique in which simply tapping the string on the fret will produce an overtone based on the string's fundamental and the node-length of the vibrating portion of the string. Variations in the technique occur when the fundamental length of the string is altered by pressing down on the string at a chosen fret with one hand and then tapping a higher fret with the other. You can also generate harmonics by simply touching the string at its node points while it is vibrating, thereby damping the fundamental, much as violinists do. And if you do so along the half of the string below the 12th fret, you can most clearly hear the overtones in the diatonic *b*7th scale of the fundamental. Some occur close to but not exactly on a fret. But of those that do you will find the octave at the 12th fret, its 5th at the 7th, the next octave at the 5th fret, and its 3rd at the 4th! So, musical, mathematical, and mechanical truth once again perfectly merge.

On p. 368 I drew the parallel between the harmonic truth of 12 tones and the cosmic truth of 12 astrological signs and months in a year. Clearly, we now see the first as much more universal and essential to existence than either of the latter, which are truths based on the observations of this planet's ancient civilizations. To once again consider the perspective of other potential inhabitants of the Universe, neither of those may matter. However, the truth of 12 revealed in the merging of music, mathematics, and mechanics surely would. And one last point in this regard must be made. Their merging is evidenced in the 12-TET scale. Yet, other TET scales were mentioned. Some in our context may seem more appropriate for non-harmonic systems, and certainly there are those throughout the Universe. Even an orchestra has its cymbals and drums. But consider the greater diversity allowed by 24-TET, common in Arabian music: 12 subdivided by 2, with a quarter-tone as its basic unit. How about 36-TET, 12 then subdivided by 3, or 48, subdivision by 4? Their still greater diversity would seem to play a role somewhere in the *Big Picture*. Yet pursue it even further. Ms. Alexjander found that 60 pitches in DNA could provide all 16 overtones in the harmonic series. This now takes the 12 intervals and subdivides them by the prime of 5, itself a TET world music contender. *Modern Physics* pointed out that 70 tones are needed to satisfy all keys. 72-TET would seem to do so quite nicely – subdivision by 6. But the power in its numerical truth must wait for this book's final pages, when we peek into the crystals of water for their hidden messages.